WAVES

1.) A wave is a disturbance that moves through a medium. (You can't have water waves without water!)

2.) A wave produced by a force that is perpendicular to the direction of propagation of the wave disturbance is called a transverse wave.

a.) An example of a transverse wave is:

i.) the wave produced when a taught string it jiggled.

i.) the wave produced when a rock is thrown into a pond.

3.) A wave produced by a force that is in the same direction as the propagating wave disturbance is called a *longitudinal wave*.

a.) An example of a longitudinal wave is sound wave.

RESONANCE

1.) A spring has one frequency at which it will vibrate. That frequency is called the ‘natural frequency’ of the spring system.

2.) Apply a force to the spring. If the frequency of the force matches the natural frequency of the spring system, the amplitude of the spring’s oscillation will become bigger and bigger.

3.) This is called *resonance*.

4.) In fact, if you apply a periodic force to system and the frequency of that force matches one of the natural frequencies of the system, the amplitude of the system’s oscillation will get bigger and bigger and you will have a *resonance* between the force and the system.

SUPERPOSITION OF WAVES

1.) If waves from two sources moving in the same medium superimpose on one another, the waves will overlay one another.

a.) If the overlay produces an additive effect, the superposition is said to be *constructive superposition*.

b.) If the overlay produces a subtractive effect, the superposition is said to be *destructive superposition*.

STANDING WAVES

A standing wave is generated when waves generated by two different sources in a system superimpose creating a single, orderly wave.

The classic example of the production of standing waves uses a rope tied rigidly to a fixed structure at both ends. If one end is jiggled so as to generate a wave, that wave will proceed down the rope (let's say it moves to the left) until it strikes the fixed end on the left whereupon it will bounce and move back toward the right. If the jiggle is periodic producing wave after wave, the "new" waves moving to the left will superimpose with the bounced waves moving to the right.

Normally, this superposition will appear chaotic with the rope jumping around somewhat chaotically. But under the *right condition*, the resulting motion will be very orderly.

STANDING WAVES on a ROPE

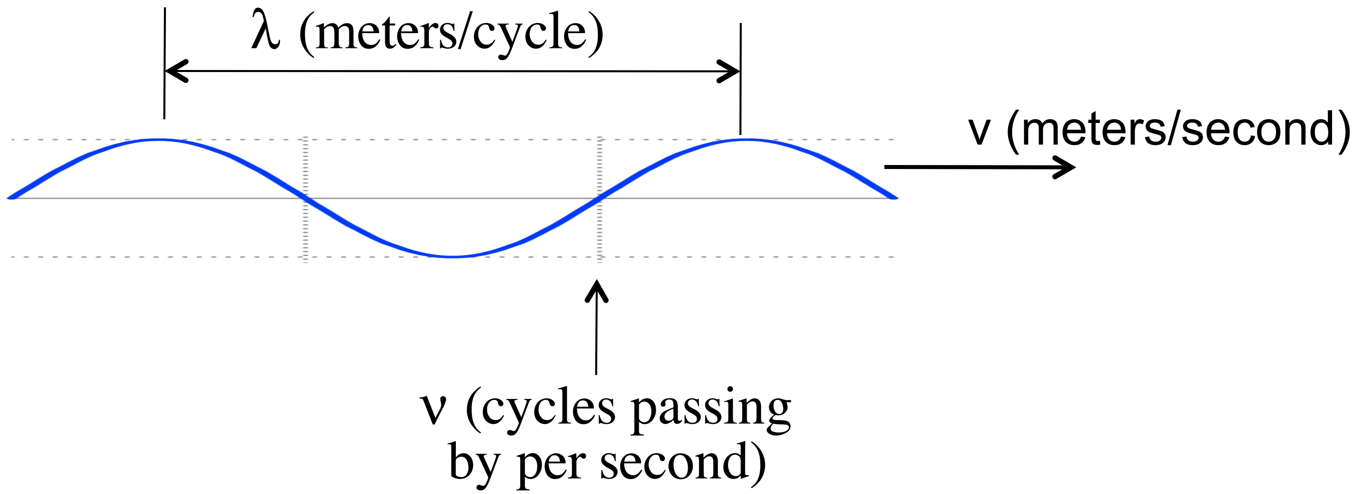
The *right condition* is essentially that of resonance:

> If the frequency of the force producing the vibration just happens to match one of the natural frequencies of the oscillating system, the resulting superposition will produce *standing waves*.

There is a trick tp determining what frequency (or frequencies) will do this. That trick starts by looking at the geometry of the system.

To see how this works, we'll look at our rope example more closely. But before we do, there are a few observation we need to make.

1.) As you know, a wave is traditionally characterized by its frequency ν, its wavelength λ and its wave velocity v.



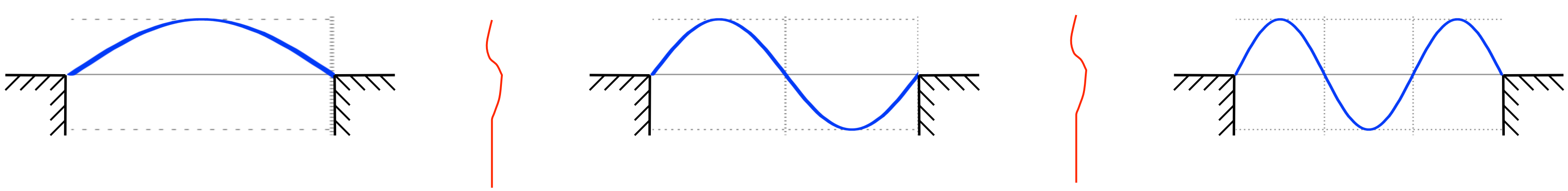
2.) The relationship between these three parameters is:

$$v = \lambda \, \nu.$$

3.) That means that if we know the wave velocity, and if we can determine the "appropriate wavelength" for a given situation, we can determine the frequency of the wave and, hence, the frequency our force must vibrate at to generate a standing wave.

4.) So how to get the "appropriate wavelength" for a given system?

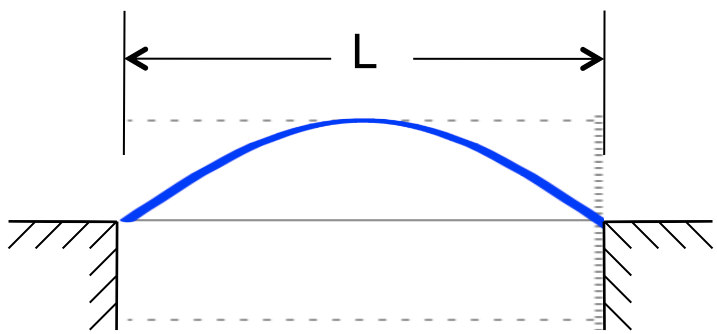
Let's look at our rope situation. What constraint must our "appropriate wavelength" satisfy? Well, it better have a node (a "fixed" point) at each end, as each end is tied to a rigid structure and can't move. What kind of wave will do that? Three are shown below.



NOTICE: In all three cases, the end-point constraints are met (that is, they ALL have nodes at both ends of their respective wavelengths).

5.) This is all fine and dandy, but how does it help with our problem? It helps because we know that the span between the ends is a fixed distance "L." All we have to do it link "L" to the wavelengths viewed, and we have the wavelengths in terms of a know quantity. Specifically:

a.) For the first situation: We know "L," so the question is, "How many wavelengths are in "L?"



Looking at the wave, we can see that there are two quarter-wavelengths ($\lambda/4$) in L (sounds obscure—you'll get used to it). That is, we can write:

$$L = 2\left(\lambda/4\right)$$
$$\Rightarrow \quad \lambda = 2L.$$

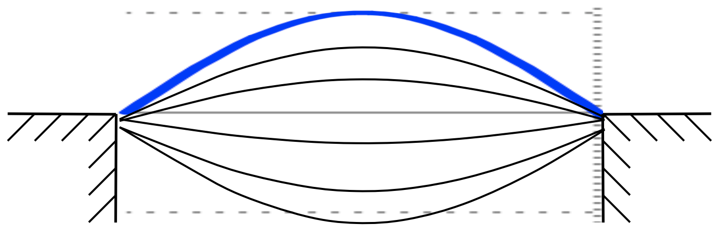
6.) Assuming we know the wave velocity (this would normally be given), we can write:

$$
\begin{aligned}
v &= \lambda\, \nu \\
&= (2L)\nu \\
\Rightarrow \quad \nu &= \frac{v}{2L}
\end{aligned}
$$

7.) So let's say the wave velocity is 3 m/s and the length of the rope is 2 meters. That means:

$$
\begin{aligned}
\nu &= \frac{v}{2L} \\
&= \frac{(3 \text{ m/s})}{2(2 \text{ m})} \\
\Rightarrow \quad \nu &= .75 \text{ sec}^{-1} \quad \text{(this unit is the same as a Hertz, Hz)}
\end{aligned}
$$

8.) Apparently, if we jiggle the rope at .75 Hz, we will get a standing wave on the rope that, over time, looks like:



9.) We could do a similar bit of analysis for the other two waveforms.

10.) ONE OTHER THING: If there had been any *internal constraints*—if, for instance, we had pinched the rope at *L/2* making that point a node, then our waveform wouldn't have worked (look at it—there is an anti-node—an extreme—at *L/2*) and we would have had to have done a bit more thinking (you'll see examples of this in class).

So in general, what is the procedure to be followed in these kinds of problems? They are:

a.) Identify the end-point constraints. (In our example, it was that there had to be nodes—fixed points—at each end.)

b.) On a sine wave, identify what the waveform is going to look like. (You'll see examples of this in class.)

c.) Once you know what the waveform should look like, be sure that any internal constraints imposed on the system are met by the waveform. (Again, you'll see examples of this in class.)

d.) When satisfied, ask the question, "How many quarter-wavelength are there in *L*?" Put a little differently, fill in the ? in the expression:

$$(?)\left(\lambda/4\right) = L$$

e.) Solve for λ in terms of *L*, then use $v = \lambda \nu$ to get the required frequency.

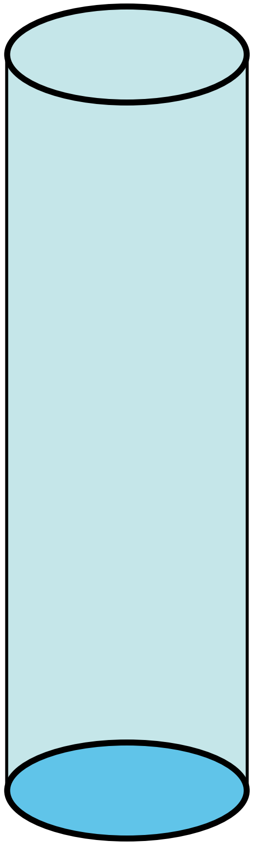
Pipe Closed at One End

Another outstanding example of a standing wave is the waveform that is generated when air is piped through a tube.

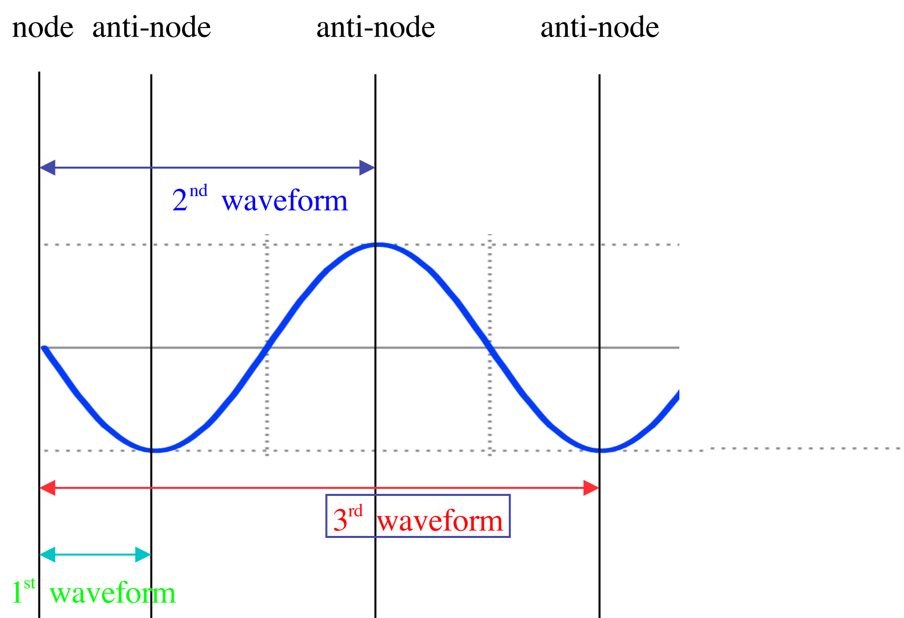
1.) Consider a pipe of length "L" closed at one end. What frequency of sound will stand in the pipe?

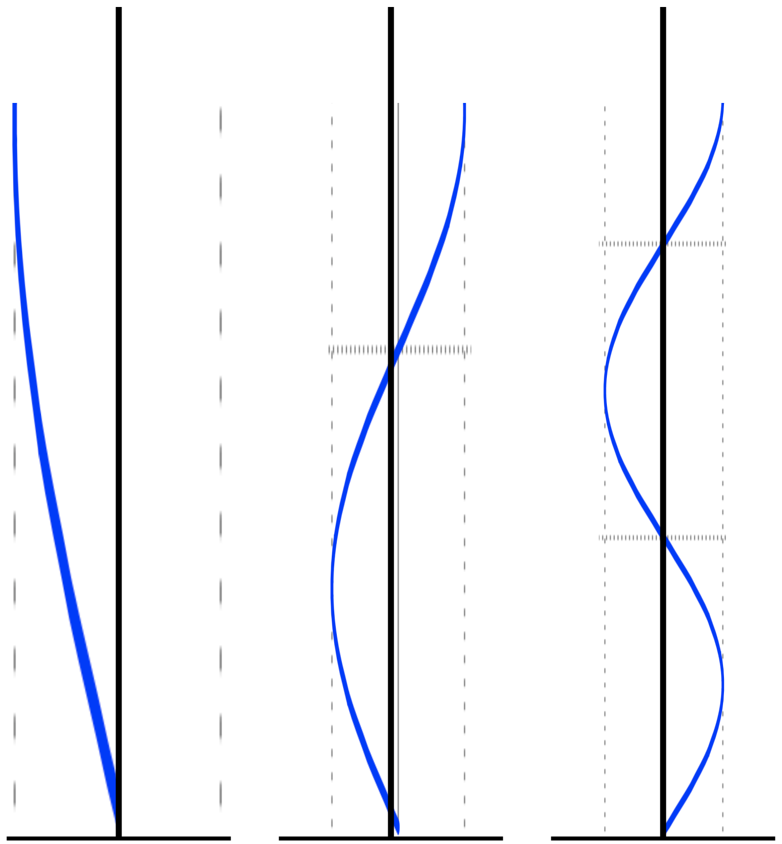
2.) In a problem like this, the first thing you have to do is identity what standing waves will fit in the pipe. To do that, you have to begin by identifying the end-point constraints.

a.) For a pipe closed at one end, the end-point constraints dictate an anti-node at the open end and a node at the closed end.



3.) The waveforms that fit the bill are shown below, then reproduced in the vertical:





4.) Each section of wave has a numerical length equal to the length of the tube, or "L."

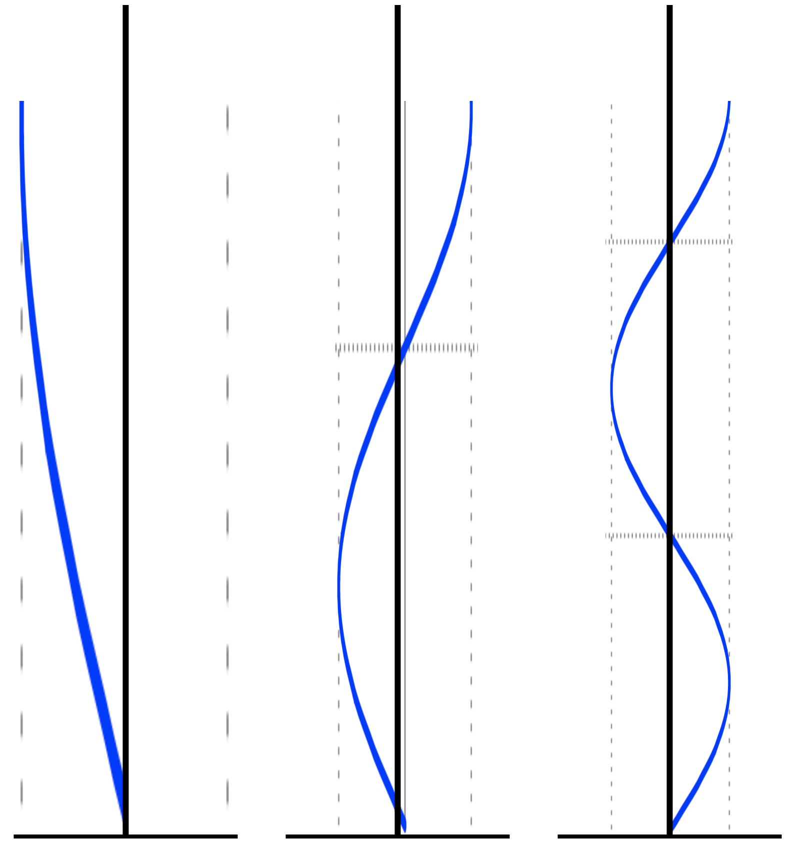
5.) If we ask the question, "How many wavelengths are there in "L?" (That is, we need to complete the

$$?\lambda = \mathrm{L}$$

By examination, there is a quarter wavelength in "L," so we can write:

$$\frac{1}{4}\lambda_1 = \mathrm{L}$$

$$\Rightarrow \quad \lambda_1 = 4\mathrm{L}$$



Minor Note: In real life, the effective length of the tube has to be altered due to perturbation effects at the ends. In the case of a singly open tube, the effective length of the tube isn't "L" but rather "L+.4d," where "d" is the tube's diameter. For doubly open tube, it's "L+.8d."

6.) We know the speed of sound in air is approximately 330 m/s and we know the relationship between a wave's velocity and its wavelength and frequency is $v = \lambda_1 \nu_1$. Assuming the tube's length is 2 meters (and ignoring the radius correction mentioned at the bottom of the previous page), we can write:

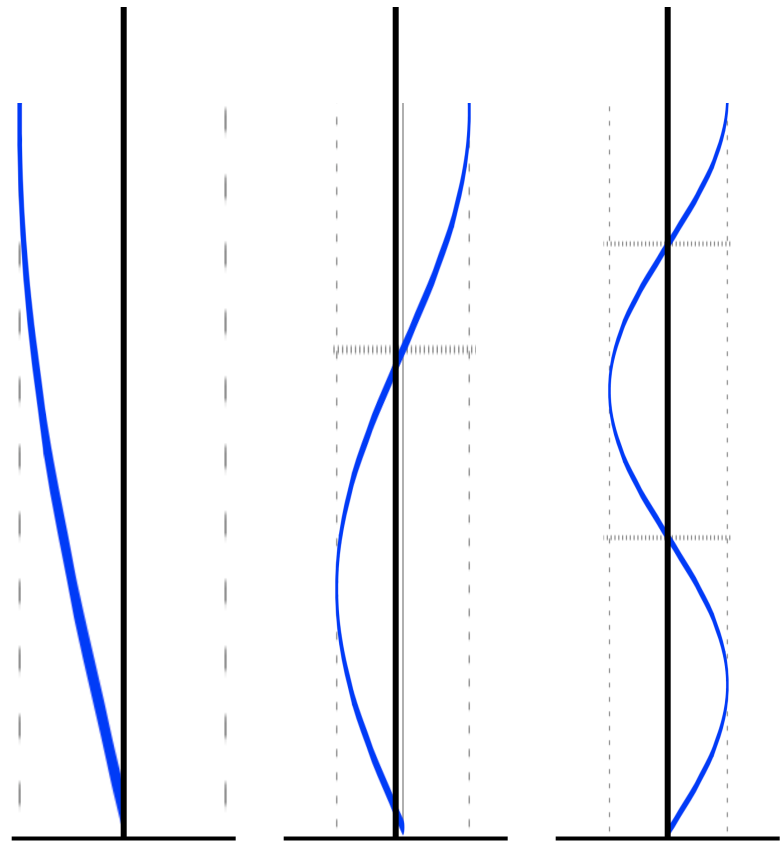
$$v = \lambda_1 \nu_1$$

$$\Rightarrow \quad \nu_1 = \frac{v}{\lambda_1}$$

$$\Rightarrow \quad \nu_1 = \frac{v}{4L}$$

$$\Rightarrow \quad \nu_1 = \frac{(330 \text{ m/s})}{4(2 \text{ m})}$$

$$\Rightarrow \quad \nu_1 = 41.25 \text{ Hz}$$



7.) Put a 41.25 Hz tuning fork at the mouth of our tube and it will howl quite loudly.

8.) Doing the same calculation for the second situation where there is three-quarter of a wave in "L," we can write:

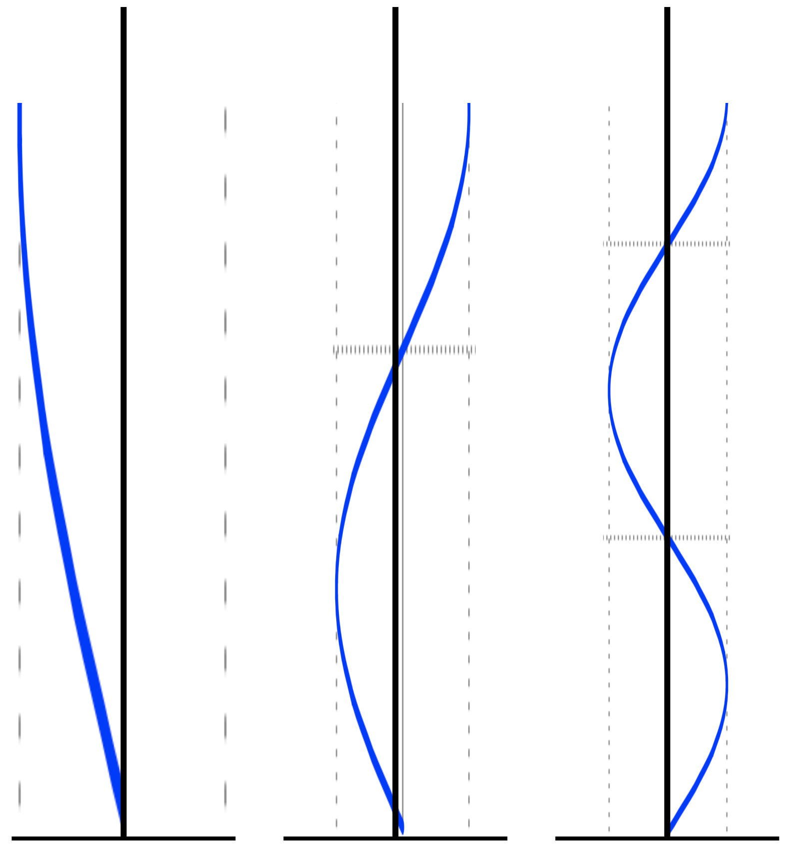
$$v = \lambda_2 \nu_2$$

$$\Rightarrow \quad \nu_2 = \frac{v}{\lambda_2}$$

$$\Rightarrow \quad \nu_2 = \frac{v}{\left(4/3\right)L}$$

$$\Rightarrow \quad \nu_2 = \frac{(3)(330 \text{ m/s})}{4(2 \text{ m})}$$

$$\Rightarrow \quad \nu_2 = 123.75 \text{ Hz}$$



9.) Put a 123.75 Hz tuning fork at the mouth of our tube and it will howl quite loudly.

10.) Doing the same calculation for the third situation where there is five-quarters of a wave in "L," we can write:

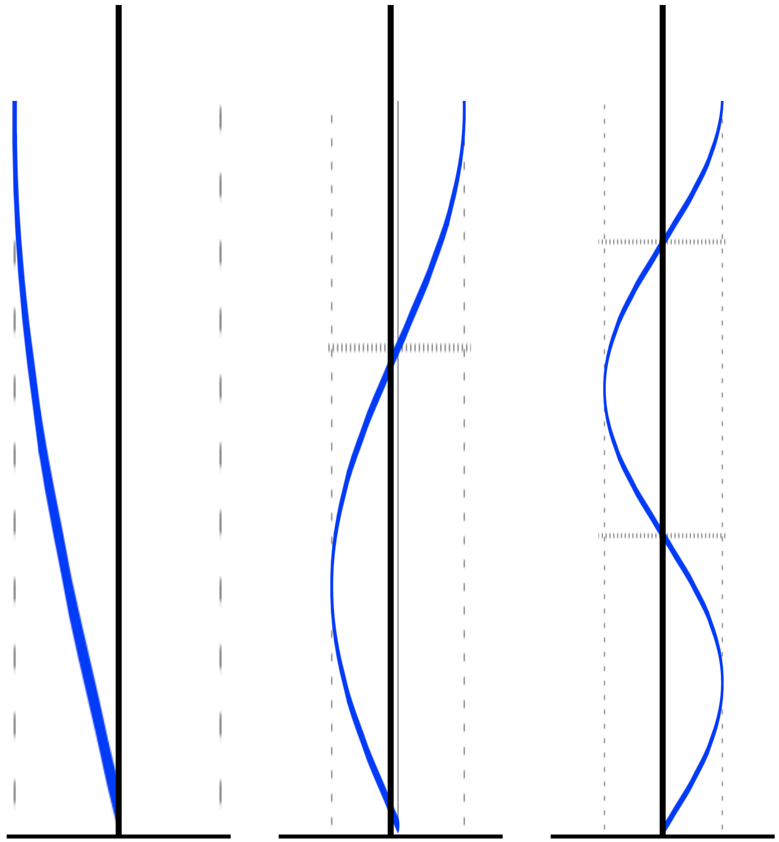
$$v = \lambda_3 \nu_3$$

$$\Rightarrow \quad \nu_3 = \frac{v}{\lambda_3}$$

$$\Rightarrow \quad \nu_3 = \frac{v}{\left(4/5\right)L}$$

$$\Rightarrow \quad \nu_3 = \frac{(5)(330 \text{ m/s})}{4(2 \text{ m})}$$

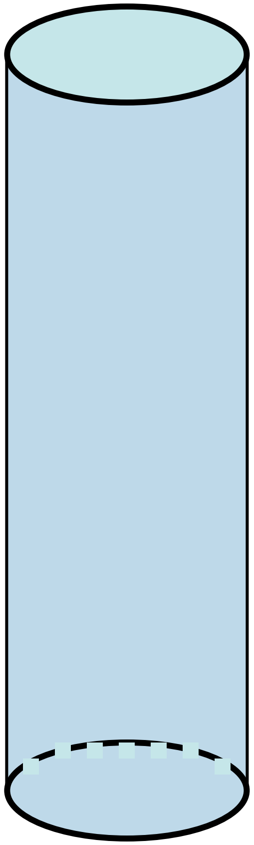
$$\Rightarrow \quad \nu_3 = 206.25 \text{ Hz}$$



11.) Put a 206.25 Hz tuning fork at the mouth of our tube and it will howl quite loudly.

Pipe Open at Both Ends

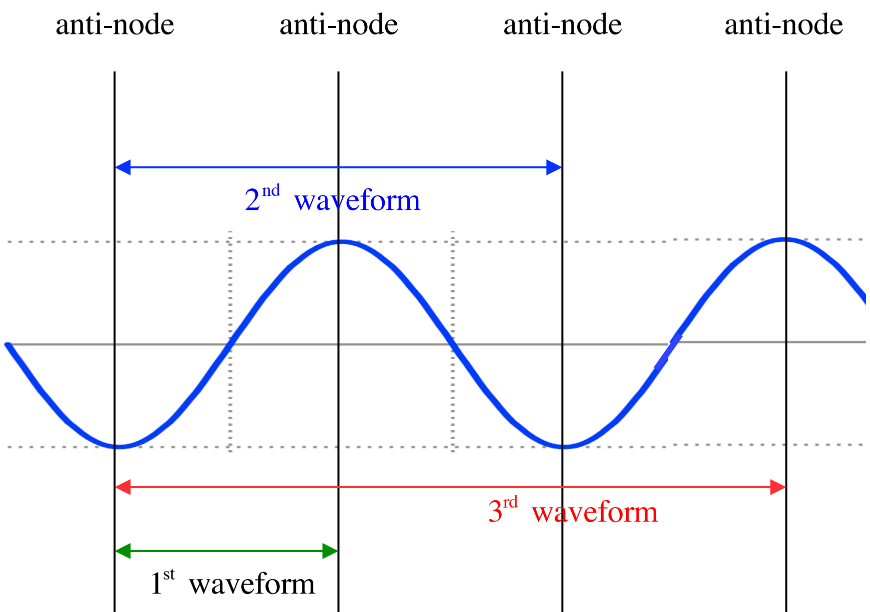
12.) Consider a pipe of length "L" open at both ends. What frequency of sound will stand in the pipe?

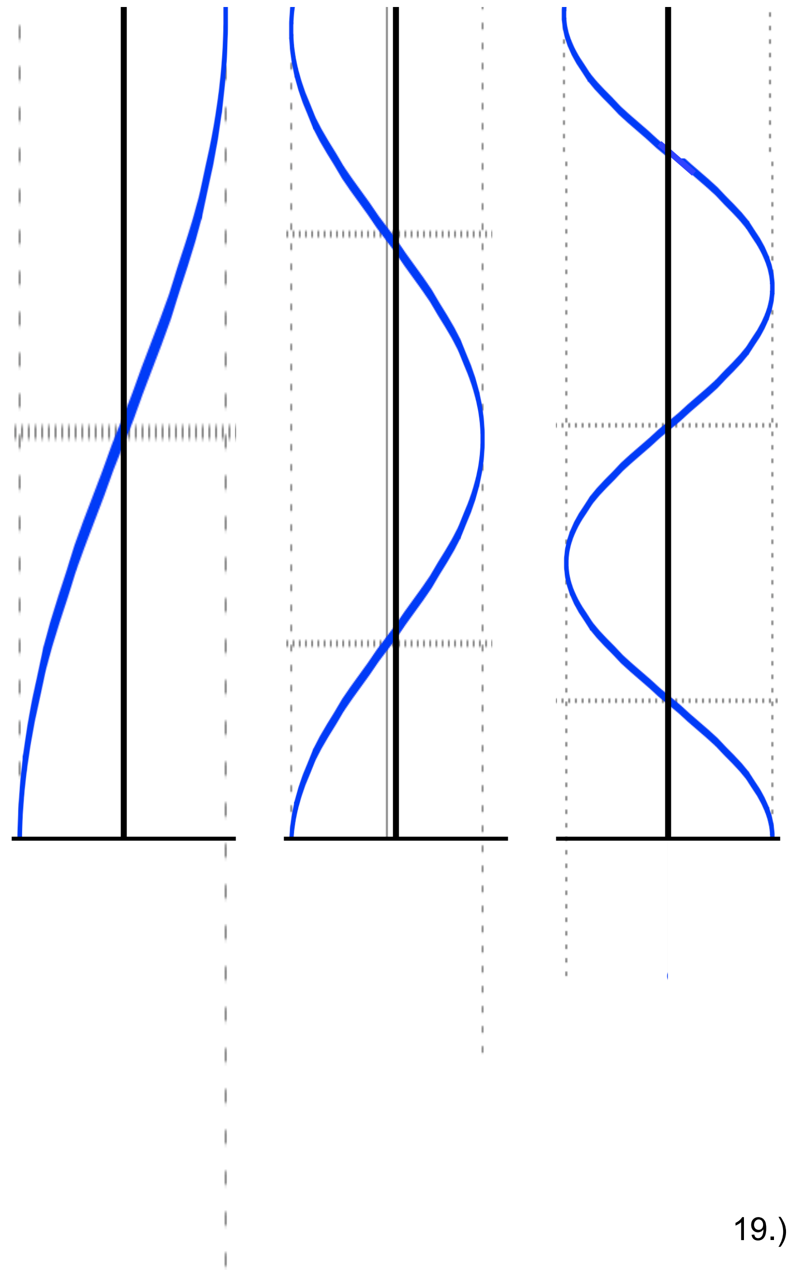


13.) In a problem like this, the first thing you have to do is identity what standing waves will fit in the pipe. To do that, you have to begin by identifying the end-point constraints.

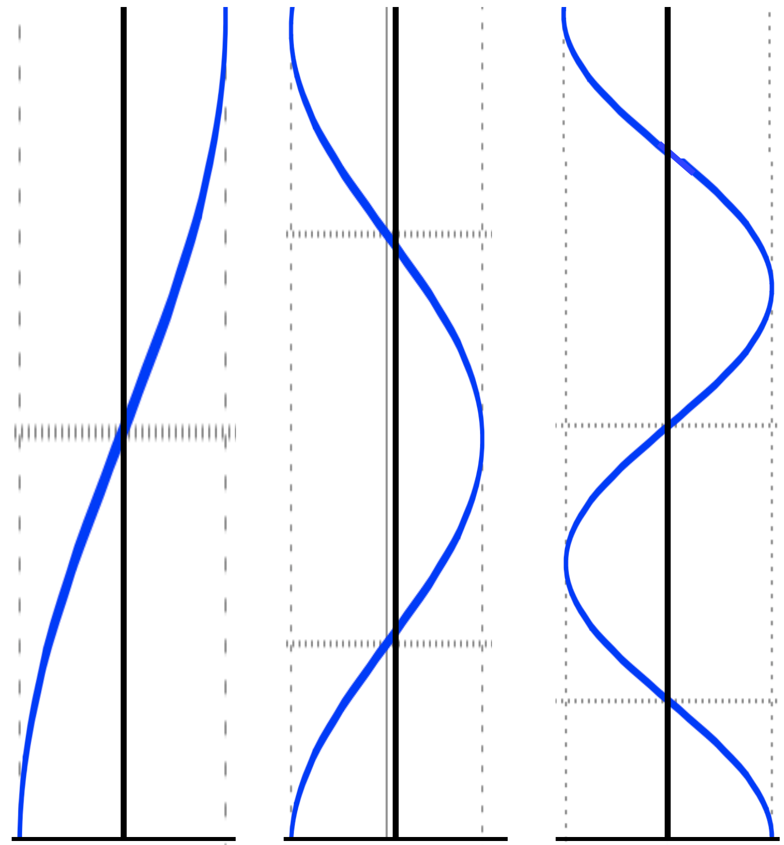
a.) For a pipe open at both ends, the end-point constraints dictate anti-nodes at the both ends.

14.) On the sine wave presented below, you can see the waveforms that satisfy the end-point constraints. Once determined, they can be put on the sketch to the right (though on a test you probably won't redraw the sketch to fit the actual pipe).





15.) The lowest frequency--the longest wavelength--is shown as the first sketch to the right. Noting that $\frac{1}{2}\lambda_1 = L$, we can write:



$$v = \lambda_1 \nu_1$$

$$\Rightarrow \quad \nu_1 = \frac{v}{\lambda_1}$$

$$\Rightarrow \quad \nu_1 = \frac{v}{2L}$$

$$\Rightarrow \quad \nu_1 = \frac{(1/2)(330 \text{ m/s})}{(2 \text{ m})}$$

$$\Rightarrow \quad \nu_1 = 82.5 \text{ Hz}$$

16.) Send a 330 Hz sound wave down the tube, and it will howl at that frequency.

16.) The second lowest frequency--the second longest wavelength--is shown as the middle sketch to the right. Noting that $\lambda_2 = L$, we can write:

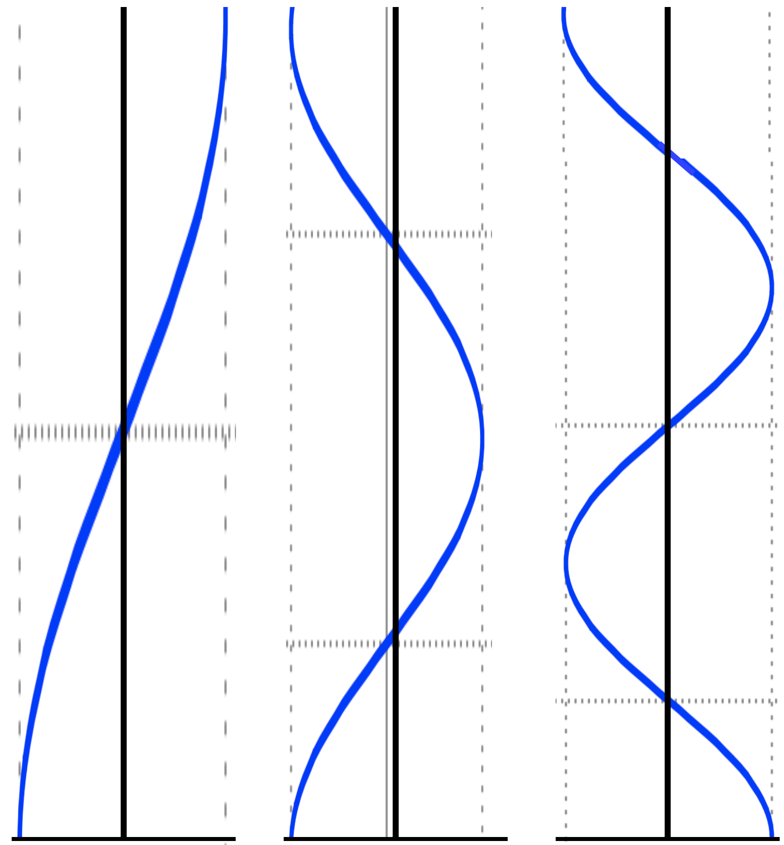
$$v = \lambda_2 \nu_2$$

$$\Rightarrow \quad \nu_2 = \frac{v}{\lambda_2}$$

$$\Rightarrow \quad \nu_2 = \frac{v}{L}$$

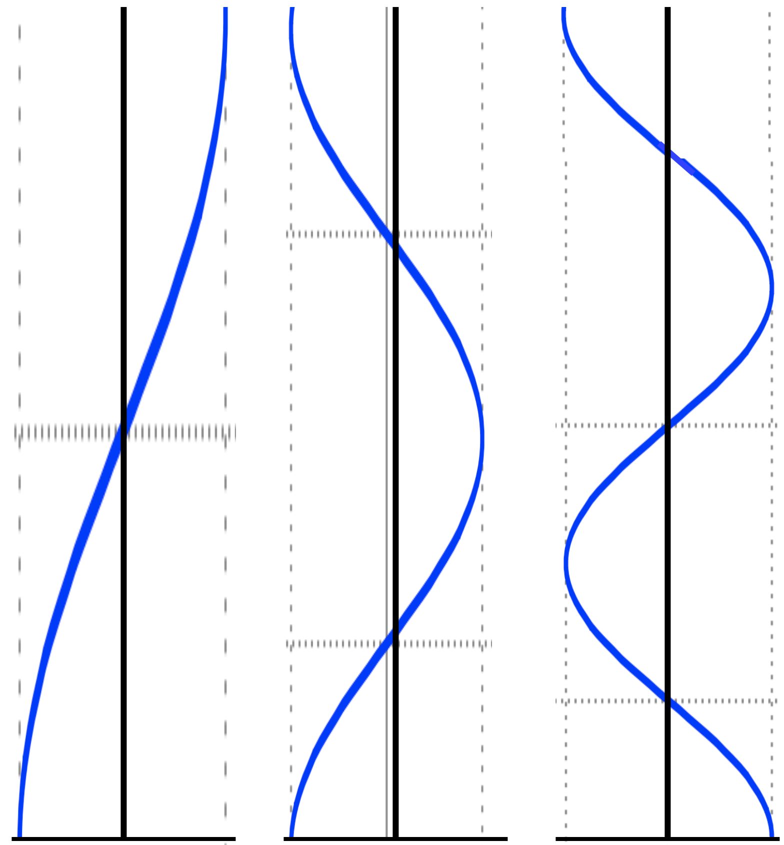
$$\Rightarrow \quad \nu_2 = \frac{(330 \text{ m/s})}{(2 \text{ m})}$$

$$\Rightarrow \quad \nu_2 = 165 \text{ Hz}$$



16.) Send a 165 Hz sound wave down the tube, and it will howl at that frequency.

16.) The second lowest frequency--the second longest wavelength--is shown as the middle sketch to the right. Noting that $\frac{6}{4}\lambda_3 = \frac{3}{2}\lambda_3 = L$, we can write:



$$v = \lambda_3 \nu_3$$

$$\Rightarrow \quad \nu_3 = \frac{v}{\lambda_3}$$

$$\Rightarrow \quad \nu_3 = \frac{v}{\left(\frac{2}{3}L\right)}$$

$$\Rightarrow \quad \nu_3 = \frac{(3)(330 \text{ m/s})}{(2)(2 \text{ m})}$$

$$\Rightarrow \quad \nu_3 = 247.5 \text{ Hz}$$

16.) Send a 110 Hz sound wave down the tube and it will howl at that frequency.